

PHI 103 - Propositional Logic
Natural Deduction (part 1)

Implication Rules
valid argument forms

Propositional Logic

Implication Rules

I. Material Implication:

A. *Modus Ponens* (*affirming the antecedent*)

B. *Modus Tollens* (*denying the consequent*)

C. **Hypothetical Syllogism** (the transitive property)

II. Disjunction:

A. **Disjunctive Syllogism** -

Propositional Logic

Implication Rules

- **Modus Ponens** (the “way” of *affirmation*)

P1) *If* dogs are mammals, *then* they have hearts.

P2) Dogs *are* mammals.

C) Therefore, dogs have hearts.

$$\begin{array}{l} \text{P1) } D \supset H \\ \text{P2) } D \\ \hline \therefore H \end{array}$$

Valid

D	H	D \supset H / D // H				
T	T	T	T	T	T	T
T	F	T	F	F	T	F
F	T	F	T	T	F	T
F	F	F	T	F	F	F

Propositional Logic

Implication Rules

- **Modus Tollens** (the “way” of *denial*)

P1) *If* cats are birds, *then* they have feathers.

P2) *It's not* the case that cats have feathers.

C) Therefore, *it's not* the case that cats are birds.

$$\begin{array}{l} \text{P1) } C \supset F \\ \text{P2) } \sim F \\ \hline \therefore \sim C \end{array}$$

Valid

C	F	C \supset F / \sim F // \sim C						
T	T	T	T	T	F	T	F	T
T	F	T	F	F	T	F	F	T
F	T	F	T	T	F	T	T	F
F	F	F	T	F	T	F	T	F

Propositional Logic

Implication Rules

- **Hypothetical Syllogism** (the *transitive property*)

P1) *If* Kato is a mammal, *then* he is warm-blooded.

P2) *If* he is warm-blooded, *then* he has a heart.

C) Therefore, *If* Kato is a mammal, *then* he has a heart.

P1) $K \supset W$

P2) $W \supset H$

$\therefore K \supset H$

Valid

p	q	r	$p \supset q$	$q \supset r$	$p \supset r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	F	F

Propositional Logic

Implication Rules

- **Disjunctive Syllogism** (the method of *elimination*)

P1) *Either* cats have feathers, *or* they have fur.

P2) *It's not* the case that cats have feathers.

C) Therefore, cats have fur.

$$\begin{array}{l} \text{P1) } C \vee F \\ \text{P2) } \sim C \\ \hline \therefore F \end{array}$$

Valid

p	q	$p \vee q \mid \sim p \parallel q$					
T	T	T	T	T	F	T	T
T	F	T	T	F	F	T	F
F	T	F	T	T	T	F	T
F	F	F	F	F	T	F	F

Practice

$$\begin{array}{l} 1. (C \vee F) \supset H \\ 2. \frac{C \vee F}{\therefore H} \quad \text{MP} \end{array}$$

$$\begin{array}{l} 1. (K \vee L) \supset W \\ 2. \frac{W \supset (H \cdot J)}{\therefore (K \vee L) \supset (H \cdot J)} \quad \text{HS} \end{array}$$

$$\begin{array}{l} 1. D \\ 2. \frac{D \supset H}{\therefore H} \quad \text{MP} \end{array}$$

$$\begin{array}{l} 1. X \supset Y \\ 2. \frac{\sim Y}{\therefore \sim X} \quad \text{MT} \end{array}$$

$$\begin{array}{l} 1. \sim M \supset (R \supset S) \\ 2. \frac{(C \supset K) \supset \sim M}{\therefore (C \supset K) \supset (R \supset S)} \quad \text{HS} \end{array}$$

$$\begin{array}{l} 1. \sim G \supset \sim (R \supset S) \\ 2. \frac{\sim \sim (R \supset S)}{\therefore \sim \sim G} \quad \text{MT} \end{array}$$

Propositional Logic

Implication Rules

- **Demonstrating Validity** (proofs in natural deduction)

1. $A \supset B$
2. $C \vee A$
3. A / B
4. B 1, 3, MP

1. $C \supset B$
2. $A \supset B$
3. $\sim B$ / $\sim A$
4. $\sim A$ 2, 3, MT

1. $S \supset T$
2. $T \supset U$
3. $R \supset S$ / $R \supset U$
4. $R \supset T$ 1, 3, HS
5. $R \supset U$ 2, 4, HS

Propositional Logic

Implication Rules

- **Demonstrating Validity** (proofs in natural deduction)

1. $A \vee B$

2. $\sim C \supset \sim A$

3. $C \supset D$

4. $\sim D$ / B

5. $\sim C$ 3, 4, MT

6. $\sim A$ 2, 5, MP

7. B 1, 6, DS

Propositional Logic

Implication Rules

- **Demonstrating Validity** (proofs in natural deduction)

1. $E \supset (K \supset L)$	
2. $F \supset (L \supset M)$	
3. $G \vee E$	
4. $\sim G$	
5. F	/ $(K \supset M)$
6. E	3, 4, DS
7. $K \supset L$	1, 6, MP
8. $L \supset M$	2, 5, MP
9. $K \supset M$	7, 8, HS

Propositional Logic

Implication Rules

I. Material Implication:

A. *Modus Ponens* (*affirming the antecedent*)

B. *Modus Tollens* (*denying the consequent*)

C. **Hypothetical Syllogism** (the transitive property)

II. Disjunction:

A. **Disjunctive Syllogism**

B. **Addition**

III. Conjunction:

A. **Conjunction**

B. **Simplification**

C. **Constructive Dilemma**

Propositional Logic

Implication Rules

- **Addition** (add *anything* you need!)

P1) Cats have feathers.

C) *Either* cats have feathers, *or* they have fur.

$$1. \frac{C}{\therefore C \vee F}$$

Valid

p	q	p	\parallel	$p \vee q$
T	T	T		T T T
T	F	T		T T F
F	T	F		F T T
F	F	F		F F F

Propositional Logic

Implication Rules

- **Conjunction** (connect with the dots)

P1) Cats have fur.

P2) Cats have whiskers.

C) Therefore, cats have fur and cats have whiskers.

$$\begin{array}{l} 1. \mathbf{C} \\ 2. \mathbf{W} \\ \hline \therefore \mathbf{C \cdot W} \end{array}$$

Valid

p	q	$p \mid q$	$q \parallel p$	$p \cdot q$
T	T	T	T	T
T	F	T	F	F
F	T	F	T	F
F	F	F	F	F

Propositional Logic

Implication Rules

- **Simplification** (*dis-connect the dots*)

P1) Cats have fur *and* cats have whiskers.

C) Therefore, cats have fur.

$$\begin{array}{l} 1. \frac{\mathbf{F} \cdot \mathbf{W}}{\therefore \mathbf{F}} \end{array}$$

Valid

p	q	$p \cdot q$			$//$	p
T	T	T	T	T		T
T	F	T	F	F		T
F	T	F	F	T		F
F	F	F	F	F		F

Propositional Logic

Implication Rules

- **Constructive Dilemma** (*concluding the consequences*)

P1) If Kato is a mammal then he is warm-blooded *and* if Kato is a dog then he has fur.

P2) Either Kato is a mammal or he's a dog.

C) Therefore, Kato is warm-blooded, *or* he has fur.

$$\begin{array}{l} 1. (\mathbf{M} \supset \mathbf{W}) \cdot (\mathbf{D} \supset \mathbf{F}) \\ 2. \mathbf{M} \vee \mathbf{D} \\ \hline \therefore \mathbf{W} \vee \mathbf{F} \end{array}$$

Propositional Logic

Implication Rules

- **Constructive Dilemma** (*concluding the consequences*)

A constructive dilemma is performing two modus Ponens simultaneously.

$$1. (p \supset q) \cdot (r \supset s)$$

$$2. p \quad \vee \quad r$$

$$\therefore q \vee s$$

$$1. (p \supset q) \cdot (r \supset s)$$

$$2. p \vee r$$

$$\therefore q \vee s$$

Propositional Logic

Implication Rules

1. $(M \supset W) \cdot (D \supset F)$

2. $M \vee D$

$\therefore W \vee F$

**CD -
Valid**

p	q	r	s	$(p \supset q) \cdot (r \supset s) / p \vee r // q \vee s$												
T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	T	F	T	F	F	T	T	T	T	T	F
T	T	F	T	T	T	T	T	F	T	T	T	T	F	T	T	T
T	T	F	F	T	T	T	T	F	T	F	T	T	F	T	T	F
T	F	T	T	T	F	F	F	T	T	T	T	T	T	F	T	T
T	F	T	F	T	F	F	F	T	F	F	T	T	T	F	F	F
T	F	F	T	T	F	F	F	F	T	T	T	T	F	F	T	T
T	F	F	F	T	F	F	F	F	T	F	T	T	F	F	F	F
F	T	T	T	F	T	T	T	T	T	T	F	T	T	T	T	T
F	T	T	F	F	T	T	F	T	F	F	F	T	T	T	T	F
F	T	F	T	F	T	T	T	F	T	T	F	F	F	T	T	F
F	T	F	F	F	T	T	T	F	T	F	F	F	F	T	T	F
F	F	T	T	F	T	F	T	T	T	T	F	T	T	F	T	T
F	F	T	F	F	T	F	F	T	F	F	F	T	T	F	F	F
F	F	F	T	F	T	F	F	T	T	T	F	T	T	F	F	T
F	F	F	F	F	T	F	T	F	T	T	F	F	F	F	T	T
F	F	F	F	F	T	F	T	F	T	F	F	F	F	F	F	F

Propositional Logic

Implication Rules

MP

$$\begin{array}{l} 1. p \supset q \\ 2. p \\ \hline \therefore q \end{array}$$

MT

$$\begin{array}{l} 1. p \supset q \\ 2. \sim q \\ \hline \therefore \sim p \end{array}$$

HS

$$\begin{array}{l} 1. p \supset q \\ 2. q \supset r \\ \hline \therefore p \supset r \end{array}$$

DS

$$\begin{array}{l} 1. p \vee q \\ 2. \sim p \\ \hline \therefore q \end{array}$$

ADD

$$\begin{array}{l} 1. p \\ \hline \therefore p \vee q \end{array}$$

Conj.

$$\begin{array}{l} 1. p \\ 2. q \\ \hline \therefore p \cdot q \end{array}$$

Simp.

$$\begin{array}{l} 1. p \cdot q \\ \hline \therefore p \end{array}$$

CD

$$\begin{array}{l} 1. (p \supset q) \cdot (r \supset s) \\ 2. p \vee r \\ \hline \therefore q \vee s \end{array}$$