PHI 103 - Propositional Logic

Natural Deduction (part 2)

Replacement Rules

logically equivalent propositions

Propositional Logic

Replacement Rules

Five Common *Logically Equivalent* Propositions

1. **Double Negative** (*DN*)

   $p :: \sim\sim p$

2. **Commutativity** (*Com*)

   $(p \cdot q) :: (q \cdot p)$
   $(p \vee q) :: (q \vee p)$

3. **Associativity** (*Assoc*)

   $[p \vee (q \vee r)] :: [(p \vee q) \vee r]$
   $[p \cdot (q \cdot r)] :: [(p \cdot q) \cdot r]$

4. **De Morgan's Rule** (DM)

   $\sim(p \cdot q) :: (\sim p \vee \sim q)$
   $\sim(p \vee q) :: (\sim p \cdot \sim q)$

5. **Distribution** (*Dist*)

   $[p \cdot (q \vee r)] :: [(p \cdot q) \vee (p \cdot r)]$
   $[p \vee (q \cdot r)] :: [(p \vee q) \cdot (p \vee r)]$

Propositional Logic

Replacement Rules

Double Negative (DN)

p :: ~ ~ p

p	*p* :: ~ ~ *p*
T	**T** T F T
F	**F** F T F

Propositional Logic

Replacement Rules

Commutativity (Comm)

$$(p \cdot q) :: (q \cdot p)$$
$$(p \vee q) :: (q \vee p)$$

p	q	$(p \cdot q)$			::	$(q \cdot p)$		
T	T	T	**T**	T		T	**T**	T
T	F	T	**F**	F		F	**F**	T
F	T	F	**F**	T		T	**F**	F
F	F	F	**F**	F		F	**F**	F

p	q	$(p \vee q)$			::	$(q \vee p)$		
T	T	T	**T**	T		T	**T**	T
T	F	T	**T**	F		F	**T**	T
F	T	F	**T**	T		T	**T**	F
F	F	F	**F**	F		F	**F**	F

Propositional Logic

Replacement Rules

Associativity (Assoc)

[p ∨ (q ∨ r)] :: [(p ∨ q) ∨ r)]

[p · (q · r)] :: [(p · q) · r)]

p q r	[*p* ∨ (*q* ∨ *r*)] :: [(*p* ∨ *q*) ∨ *r*]									
T T T	T	**T**	T T T		T	T T	**T**	T
T T F	T	**T**	T T F		T	T T	**T**	F
T F T	T	**T**	F T T		T	T F	**T**	T
T F F	T	**T**	F F F		T	T F	**T**	F
F T T	F	**T**	T T T		F	T T	**T**	T
F T F	F	**T**	T T F		F	T T	**T**	F
F F T	F	**T**	F T T		F	T F	**T**	T
F F F	F	**F**	F F F		F	F F	**F**	F

p q r	[*p* · (*q* · *r*)] :: [(*p* · *q*) · *r*]									
T T T	T	**T**	T T T		T	T T	**T**	T
T T F	T	**F**	T F F		T	T T	**F**	F
T F T	T	**F**	F F T		T	F F	**F**	T
T F F	T	**F**	F F F		T	F F	**F**	F
F T T	F	**F**	T T T		F	F T	**F**	T
F T F	F	**F**	T F F		F	F T	**F**	F
F F T	F	**F**	F F T		F	F F	**F**	T
F F F	F	**F**	F F F		F	F F	**F**	F

Propositional Logic

Replacement Rules

De Morgan's Rule (DM)

~ (p · q) :: (~ p ∨ ~ q)

~ (p ∨ q) :: (~ p · ~ q)

p	*q*	~ (*p* · *q*)	::	(~ *p*	∨	~ *q*)
T	T	**F** T T T		F T	**F**	F T
T	F	**T** T F F		F T	**T**	T F
F	T	**T** F F T		T F	**T**	F T
F	F	**T** F F F		T F	**T**	T F

p	*q*	~ (*p* ∨ *q*)	::	(~ *p*	·	~ *q*)
T	T	**F** T T T		F T	**F**	F T
T	F	**F** T T F		F T	**F**	T F
F	T	**F** F T T		T F	**F**	F T
F	F	**T** F F F		T F	**T**	T F

Propositional Logic

Replacement Rules

Distribution (Dist)

[p · (q ∨ r)] :: [(p · q) ∨ (p · r)]

p q r	[*p* · (*q* ∨ *r*)] :: [(*p* · *q*) ∨ (*p* · *r*)]
T T T	T **T** T T T T T T **T** T T T
T T F	T **T** T T F T T T **T** T F F
T F T	T **T** F T T T F F **T** T T T
T F F	T **F** F F F T F F **F** T F F
F T T	F **F** T T T F F T **F** F F T
F T F	F **F** T T F F F T **F** F F F
F F T	F **F** F T T F F F **F** F F T
F F F	F **F** F F F F F F **F** F F F

Propositional Logic

Replacement Rules

Distribution (Dist)

[p ∨ (q · r)] :: [(p ∨ q) · (p ∨ r)]

p q r	[*p* ∨ (*q* · *r*)]	::	[(*p* ∨ *q*) · (*p* ∨ *r*)]
T T T	T **T** T T T		T T T **T** T T T
T T F	T **T** T F F		T T T **T** T T F
T F T	T **T** F F T		T T F **T** T T T
T F F	T **T** F F F		T T F **T** T T F
F T T	F **T** T T T		F T T **T** F T T
F T F	F **F** T F F		F T T **F** F F F
F F T	F **F** F F T		F F F **F** F T T
F F F	F **F** F F F		F F F **F** F F F

Propositional Logic

Replacement Rules

1. **Double Negative** (*DN*)

   $p :: \sim\sim p$

2. **Commutativity** (*Com*)

   $(p \cdot q) :: (q \cdot p)$
   $(p \vee q) :: (q \vee p)$

3. **Associativity** (*Assoc*)

   $[p \vee (q \vee r)] :: [(p \vee q) \vee r]$
   $[p \cdot (q \cdot r)] :: [(p \cdot q) \cdot r]$

4. **De Morgan's Rule** (DM)

   $\sim(p \cdot q) :: (\sim p \vee \sim q)$
   $\sim(p \vee q) :: (\sim p \cdot \sim q)$

5. **Distribution** (*Dist*)

   $[p \cdot (q \vee r)] :: [(p \cdot q) \vee (p \cdot r)]$
   $[p \vee (q \cdot r)] :: [(p \vee q) \cdot (p \vee r)]$

Propositional Logic

Replacement Rules

Five (*More*) Common Logically Equivalent Propositions

1. **Tautology** (*Taut*)

   $p :: (p \vee p)$
   $p :: (p \cdot p)$

2. **Transposition** (*Trans*)

   $(p \supset q) :: (\sim q \supset \sim p)$

3. **Material Implication** (*Impl*)

   $(p \supset q) :: (\sim p \vee q)$

4. **Exportation** (*Exp*)

   $[(p \cdot q) \supset r] :: [p \supset (q \supset r)]$

5. **Material Equivalence** (*Equiv*)

   $(p \equiv q) :: [(p \supset q) \cdot (q \supset p)]$
   $(p \equiv q) :: [(p \cdot q) \vee (\sim p \cdot \sim q)]$

Propositional Logic

Replacement Rules

Tautology (Taut)

p :: (p ∨ p)
p :: (p · p)

p	*p* :: *p* ∨ *p*
T	**T** T **T** T
F	**F** F **F** F

p	*p* :: *p* · *p*
T	**T** T **T** T
F	**F** F **F** F

Propositional Logic

Replacement Rules

Transposition (Trans)

(p ⊃ q) :: (~ q ⊃ ~ p)

p	*q*	*p* ⊃ *q*	::	~ *q*	⊃	~ *p*
T	T	T **T** T		F T	**T**	F T
T	F	T **F** F		T F	**F**	F T
F	T	F **T** T		F T	**T**	T F
F	F	F **T** F		T F	**T**	T F

Propositional Logic

Replacement Rules

Material Implication (Impl)

(p ⊃ q) :: (~ p ∨ q)

p	*q*	*p* ⊃ *q*	::	~ *p*	∨	*q*
T	T	T **T** T		F T	**T**	T
T	F	T **F** F		F T	**F**	F
F	T	F **T** T		T F	**T**	T
F	F	F **T** F		T F	**T**	F

Propositional Logic

Replacement Rules

Exportation (Exp)

[(p · q) ⊃ r] :: [p ⊃ (q ⊃ r)]

p q r	[(*p* · *q*) ⊃ *r*]	::	[*p* ⊃ (*q* ⊃ *r*)]
T T T	T T T **T** T		T **T** T T T
T T F	T T T **F** F		T **F** T F F
T F T	T F F **T** T		T **T** F T T
T F F	T F F **T** F		T **T** F T F
F T T	F F T **T** T		F **T** T T T
F T F	F F T **T** F		F **T** T F F
F F T	F F F **T** T		F **T** F T T
F F F	F F F **T** F		F **T** F T F

Propositional Logic

Replacement Rules

Material Equivalence (Equiv)

(p ≡ q) :: [(p ⊃ q) · (q ⊃ p)]

(p ≡ q) :: [(p · q) ∨ (~p · ~q)]

p	*q*	*p* ≡ *q* :: [(*p* ⊃ *q*) · (*q* ⊃ *p*)
T	T	T **T** T T T T T T
T	F	T **F** F T F F F T T
F	T	F **F** T F T T F F F
F	F	F **T** F F T F T T F

p	*q*	*p* ≡ *q* :: [(*p* · *q*) ∨ (~ *p* · ~ *q*)
T	T	T **T** T T T T F T F F T
T	F	T **F** F T F F F T F T F
F	T	F **F** T F F T F F F T
F	F	F **T** F F F T T F T T F

Replacement Rules

1. **Double Negative** (DN)

   $p :: \sim\sim p$

2. **Commutativity** (Com)

   $(p \cdot q) :: (q \cdot p)$
   $(p \vee q) :: (q \vee p)$

3. **Associativity** (Assoc)

   $[p \vee (q \vee r)] :: [(p \vee q) \vee r)]$
   $[p \cdot (q \cdot r)] :: [(p \cdot q) \cdot r)]$

4. **De Morgan's Rule** (DM)

   $\sim(p \cdot q) :: (\sim p \vee \sim q)$
   $\sim(p \vee q) :: (\sim p \cdot \sim q)$

5. **Distribution** (Dist)

   $[p \cdot (q \vee r)] :: [(p \cdot q) \vee (p \cdot r)]$
   $[p \vee (q \cdot r)] :: [(p \vee q) \cdot (p \vee r)]$

6. **Tautology** (Taut)

   $p :: (p \vee p)$
   $p :: (p \cdot p)$

7. **Transposition** (Trans)

   $(p \supset q) :: (\sim q \supset \sim p)$

8. **Material Implication** (Impl)

   $(p \supset q) :: (\sim p \vee q)$

9. **Exportation** (Exp)

   $[(p \cdot q) \supset r] :: [p \supset (q \supset r)]$

10. **Material Equivalence** (Equiv)

    $(p \equiv q) :: [(p \supset q) \cdot (q \supset p)]$
    $(p \equiv q) :: [(p \cdot q) \vee (\sim p \cdot \sim q)]$