PHI 103 - Propositional Logic
Lecture 2

Truth Tables

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

- **Truth *Function*** - the truth-value of any compound proposition determined solely by the truth-value of its components.

- **Statement *Variable*** - a variable that represents *any* proposition (by convention we use lower-case letters '*p*', '*q*', '*r*', '*s*', etc.).

- **Truth *Table*** - a calculation matrix used to demonstrate all logically possible truth-values of a given proposition.

Let any statement be represented by *p*:

p	*p*
T	**T**
F	**F**

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

- **Truth *Function*** - the truth-value of any compound proposition determined solely by the truth-value of its components.
- **Statement *Variable*** - any variable that represents a proposition (by convention we use lower-case letters '*p*', '*q*', '*r*', '*s*', etc.).
- **Truth *Table*** - a calculation matrix used to demonstrate all logically possible truth-values of a given proposition.

This is the negation of any proposition *p*.

Negation '~'

"opposite truth values"

p	~ *p*
T	**F**
F	**T**

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

Conjunction ‘ · ’

p	q	$p \cdot q$

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

Conjunction ‘ · ’

number of lines equals 2 to the power of the number of propositions

$$L=2^n$$

p	q	$p \cdot q$

?

$$2^2 = 4$$

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

Conjunction ‘ · ’

“both are true”

p	q	$p \cdot q$
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**F**

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

Disjunction ‘ ∨ ’

“at least one is true”

p	q	$p \vee q$
T	T	**T**
T	F	**T**
F	T	**T**
F	F	**F**

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

Material Equivalence ‘ ≡ ’

“equivalent truth values”

p	q	$p \equiv q$
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**T**

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

Material Implication ‘ ⊃ ’

*“in all cases **except** when the antecedent is true and the consequent is false”*

p	q	$p \supset q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

Propositional Logic

Truth Tables *Part 1* - Truth Functions for Logical Operators

Material Implication ‘ ⊃ ’

*“in all cases **except** when the antecedent is true and the consequent is false”*

“If you get an A on the final exam, then you will pass Logic.”

(*Getting an A on the final is **sufficient** to pass Logic.*)

“I got an A on the final! Why did you fail me?”

“***If*** you get an A on the final exam, ***then*** you will pass Logic.”

T ***F***

Propositional Logic

Truth Tables *Part 2* - Truth Tables for Compound Propositions

Method 1.a - ***if*** you already know the truth-values of the components (the *long method*)

suppose **A**, **B**, and **C** are ***true***, and **D**, **E**, and **F** are ***false***

$$(A \cdot D) \vee F$$

$$(\mathbf{T} \cdot \mathbf{F}) \vee \mathbf{F}$$

$$\mathbf{F} \quad \vee \mathbf{F}$$

$$\textcircled{\mathbf{F}}$$

Propositional Logic

Truth Tables *Part 2* - Truth Tables for Compound Propositions

Method 1.b - if you already know the truth-values of the components (the *short method*)

suppose **A**, **B**, and **C** are ***true***, and **D**, **E**, and **F** are ***false***

$$(A \cdot D) \vee F$$

T F F F F

Propositional Logic

Truth Tables *Part 3* - Truth Tables for Compound Propositions

Method 2 - if you ***do not*** know the actual truth value of the components, solve for *all possible worlds*

	A	D	F	(A · D)	v	F
line 1	T	T	T	T		T
line 2	T	T	F	T		T
line 3	T	F	T	F		T
line 4	T	F	F	F		F
line 5	F	T	T	F		T
line 6	F	T	F	F		F
line 7	F	F	T	F		T
line 8	F	F	F	F		F

$$L=2^3$$

Propositional Logic

Logical Status of Propositions

I. **Three types of Logical statements**:

A. **Logically *True*** (*tautologies*) - **always** true

B. **Logically *False*** (*self-contradiction*) - **always** false

C. **Logically *Contingent*** (truth-value *dependent*) - **sometimes** true/false

Propositional Logic

Logical Status of Propositions

Tautology (*logically true*)

"every case is true"

p	$p \vee \sim p$
T	T **T** F T
F	F **T** T F

Propositional Logic

Logical Status of Propositions

Self- Contradiction (*logically false*)

"every case is false"

p	$p \cdot \sim p$
T	T **F** F T
F	F **F** T F

Propositional Logic

Logical Status of Propositions

Contingent (*logically dependent*)

"variable truth-value"

p	q	$p \cdot q$
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**F**

Propositional Logic

Logical Relationships Between Propositions

II. **Logical Relations *between* two statements**:

A. **Contradictory** - *opposite* truth-values

p	*q*	(p ⊃ q)			≠	(p • ~ q)			
T	T	T	**T**	T		T	**F**	F	T
T	F	T	**F**	F		T	**T**	T	F
F	T	F	**T**	T		F	**F**	F	T
F	F	F	**T**	F		F	**F**	T	F

Propositional Logic

Logical Relationships Between Propositions

II. **Logical Relations *between* statements**:

A. **Contradictory** - *opposite* truth-values

B. **Equivalent** - *identical* truth-values

p	*q*	(p ⊃ q)	::	(~q ⊃ ~ p)
T	T	T **T** T		F ~~T~~ **T** F ~~T~~
T	F	T **F** F		T ~~F~~ **F** F ~~T~~
F	T	F **T** T		F ~~T~~ **T** T ~~F~~
F	F	F **T** F		T ~~F~~ **T** T ~~F~~

Propositional Logic

Logical Relationships Between Propositions

II. **Logical Relations *between* statements**:

A. **Contradictory** - *opposite* truth-values

B. **Equivalent** - *identical* truth-values

C. **Consistent** - *at least one* common (***true***) truth-value

p	*q*	(p ∨ q)			≈	(p · q)		
T	T	T	**T**	T		T	**T**	T
T	F	T	**T**	F		T	**F**	F
F	T	F	**T**	T		F	**F**	T
F	F	F	**F**	F		F	**F**	F

Propositional Logic

Logical Relationships Between Propositions

II. **Logical Relations *between* statements**:

A. **Contradictory** - *opposite* truth-values

B. **Equivalent** - *identical* truth-values

C. **Consistent** - *at least one* common (***true***) truth-value

D. **Inconsistent** - *no* common (***true***) truth-value

p	*q*	(p ≡ q)		≠	(p · ~ q)			
T	T	T	**T**	T	T	**F**	F	~~T~~
T	F	T	**F**	F	T	**T**	T	~~F~~
F	T	F	**F**	T	F	**F**	F	~~T~~
F	F	F	**T**	F	F	**F**	T	~~F~~

Propositional Logic

Logical Relationships Between Propositions

I. **Three types of Logical statements**:

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II. **Logical Relations *between* statements**:

A. **Contradictory** - *opposite* truth-values

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C. **Consistent** - *at least one* common (***true***) truth-value

D. **Inconsistent** - *no* common (***true***) truth-value

Propositional Logic

Truth Tables For Arguments

III. **Testing Arguments for Validity**:

A. ***Symbolize*** the argument

B. ***Separate*** each proposition (premises and conclusion) with slashes on a single one

C. ***Fill in*** truth values (just as you would for a compound proposition)

D. ***Look*** for a line with **true** premises and **false** conclusion

Propositional Logic

Truth Tables For Arguments

III. **Testing Arguments for Validity**:

A. ***Symbolize***

B. ***Separate***

C. ***Fill in***

D. ***Look***

If ***m***inors who commit murder are equally responsible for their crimes as adults, they should receive the ***d***eath penalty. But they're *not*. So, minors should *not* receive the ***d***eath penalty.

Invalid

(*denying the antecedent*)

M	D	M ⊃ D	/ ~M	// ~D
T	T	T **T** T	**F** ~~T~~	**F** ~~T~~
T	F	T **F** F	**F** ~~T~~	**T** ~~F~~
F	T	F **T** T	**T** ~~F~~	**F** ~~T~~
F	F	F **T** F	**T** ~~F~~	**T** ~~F~~

Propositional Logic

Truth Tables For Arguments

III. **Testing Arguments for Validity**:

A. ***Symbolize***

B. ***Separate***

C. ***Fill in***

D. ***Look***

If ***m***inors who commit murder are equally responsible for their crimes as adults, they should receive the ***d***eath penalty. And, they are. So, minors should receive the ***d***eath penalty.

Valid

(***modus ponens***)

M	D	M ⊃ D	/	M	//	D
T	T	T **T** T		T		T
T	F	T **F** F		T		F
F	T	F **T** T		F		T
F	F	F **T** F		F		F

Propositional Logic

Truth Tables For Arguments

III. **Testing Arguments for Validity**:

A. ***Symbolize*** If ***m***inors who commit murder are equally responsible for their crimes as adults, they should receive the ***d***eath penalty. But it's not the case that they deserve the ***d***eath penalty. So, ***m***inors are *not* equally responsible for their crimes.

B. ***Separate***

C. ***Fill in***

D. ***Look***

Valid

(***modus tollens***)

M	D	M ⊃ D	/ ~ D	// ~ M
T	T	T **T** T	**F** ~~T~~	**F** ~~T~~
T	F	T **F** F	**T** ~~F~~	**F** ~~T~~
F	T	F **T** T	**F** ~~T~~	**T** ~~F~~
F	F	F **T** F	**T** ~~F~~	**T** ~~F~~

Propositional Logic

Truth Tables For Arguments

III. **Testing Arguments for Validity**:

A. ***Symbolize***

B. ***Separate***

C. ***Fill in***

D. ***Look***

Either ***m***inors who commit murder are equally responsible for their crimes as adults, or they should receive the ***d***eath penalty. But it's not the case that they are equally responsible. So, they should receive the ***d***eath penalty.

Valid

(***disjunctive syllogism***)

M	D	M ∨ D	/ ~ M	// D
T	T	T **T** T	**F** ~~T~~	T
T	F	T **T** F	**F** ~~T~~	F
F	T	F **T** T	**T** ~~F~~	T
F	F	F **F** F	**T** ~~F~~	F

Propositional Logic

Truth Tables For Arguments

If ***m***inors who commit murder are equally responsible for their crimes as adults, then they should receive the ***d***eath penalty. If they should receive the ***d***eath penalty, then ***w***e should feel sorry for them. So, if ***m***inors who commit murder are equally responsible, then ***w***e should feel sorry for them.

Valid

(***hypothetical syllogism***)

M	D	W	M ⊃ D	D ⊃ W	//	M ⊃ W
T	T	T	T	T		T
T	T	F	T	F		F
T	F	T	F	T		T
T	F	F	F	T		F
F	T	T	T	T		T
F	T	F	T	F		F
F	F	T	T	T		T
F	F	F	T	T		T